ABOUT A NEW CONDITIONAL REASONING METHOD

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Received: *14 Januaryr 2026* ***Revised:*** *22 February 2026* ***Accepted:*** *5 March 2025*

UOT: 004.89 **DOİ:**https://doi.org/10.32010/ZZAF4564

Abstract: In today's complex and data-rich environment, decision-making systems often need to effectively handle information that is uncertain, imprecise, and partially reliable. Traditional logic systems, built on binary values such as true or false, are inadequate for modeling such real-world complexities. Although fuzzy logic and probabilistic reasoning have made progress in representing imprecision and uncertainty, these approaches typically address these issues separately and cannot fully capture the dual nature of imperfect information - both its imprecision and degree of reliability.This gap has driven research into Z-numbers and Z-fuzzy relations; this approach unifies imprecision and reliability within a single mathematical framework. While theoretical foundations and practical applications exist in fields like control systems, medicine, decision making and data analysis, fully developed and systematic methodologies for approximate reasoning based on Z-rules remain insufficient. Current approaches often face limitations in handling bimodal information-based rule bases. Considering these challenges, there is a need for new formal models that enable reasoning from Z-number-valued information, facilitating more human-like understanding and better management of uncertainty. The above highlights the analysis of existing fuzzy and probabilistic implication tools, the elimination of their shortcomings, and justifies the need to develop approximate reasoning methods based on Z- number conception.

Approximate reasoning plays a crucial role in decision-making and expert systems, especially in situations where information is imprecise and uncertain. In this context, fuzzy implication models serve as a key mechanism for deriving conclusions based on conditional rules. While classical logic expresses "if... then..." statements rigidly, fuzzy logic enables more flexible and human-like interpretation through fuzzy implications. However, existing fuzzy implications - such as those proposed by Mamdani, Gödel, Lukasiewicz, Zadeh, Aliev, Reichenbach, Kleene-Dienes, Goguen, Yager, Weber, Fodor and others face limitations in practical applications. These implications consider only imprecision, but fail to adequately address important aspects such as the reliability or degree of confidence in the information.

Therefore, there is a growing need for new approaches that extend the functional capabilities of fuzzy implication by extending it to Z-number-based implications. Z-numbers allow for the simultaneous modeling of both the vagueness of information and its trustworthiness (reliability or confidence). This leads to a more expressive and realistic reasoning results.

The actuality of this work lies in the fact that it investigates the shortcomings of existing fuzzy implication models, analyzes their current application potential, and justifies the development of new implications based on Z-numbers. This represents a timely and significant research direction, both theoretically and practically, for advancing modern fuzzy reasoning systems. The above-mentioned points determine the relevance of the work.

It should be noted that conditional reasoning method is the foundation of control systems and decision-making systems. This article proposes a new Z-conditional reasoning method using fuzzy and probabilistic implication.

Keywords: fuzzy implication, probabilistic implication, Z-implication, probabilty distribution, Z-number, Fuzzy number, new conditional reasonin method

Introduction

The relationship between the probabilities of occurrence of one or more dependent events is expressed using probabilistic implication. Classical logic differs from probabilistic logic. In classical logic, a true or false value is used, while in probabilistic implication, a probability value is used. Recently, the application of this concept has also been encountered in works related to the application of fuzzy theory. This can be explained by the fact that probabilistic implication can support decision-making with inaccurate information and is useful for modeling human behavior, since in this case there is incomplete information. On the other hand, it can also better model natural phenomena. There are numerous approaches in the scientific literature related to the modeling of processes characterized by imperfect information, and they are based on Professor Zadeh's Z-number theory. These approaches allow the combined use of probabilistic and fuzzy implication. However, the concept of implication is more widely used in problems requiring logical deduction. Unfortunately, these implications are used separately either in a fuzzy or probabilistic form, or in an approximated form, which leads to information loss and, again, modeling with incomplete information.

A review of existing conditional reasoning approaches reveals that most of these methods face significant challenges in effectively modeling the key characteristics of real-world problems, namely, uncertainty and imprecisionsimultaneously. Classical logic and probabilistic reasoning systems typically focus on one of these aspects, and thus fall short when it comes to processing vague, linguistically-expressed knowledge that is characteristic of human thinking.

Traditional approaches often require information and conditions to be precise and clearly defined. However, in practice, expert knowledge and real-world observations are frequently vague and subjective. These systems tend to treat all rules as equally reliable, without accounting for the degree of trust that stems from human perception, experience, or knowledge. In reality, such trust is inherently partial and should be modeled as a fuzzy value of a probability measure.

Moreover, many conditional reasoning systems lack contextual sensitivity,they apply rules uniformly, regardless of the specific situation. This reduces their alignment with human reasoning. The expressive capacity of classical and some fuzzy implication operators is also limited, as they fail to adequately reflect partial confidence or degrees of belief.

Another major limitation arises when dealing with complex systems that involve multiple interacting rules. In such cases, existing methods may produce inconsistent results or become computationally intensive. Additionally, mechanisms for prioritizing rules or resolving conflicts are either underdeveloped or entirely absent.

As a result, current conditional reasoning approaches are unable to fully capture the uncertainty and imprecision inherent in real-world information, which limits their applicability in many domains. This highlights the need for more expressive and flexible formal models.

As can be seen, the paradigm of approximate reasoning is closely linked to the nature of imperfect information. The imperfection of real-world data is mainly characterized by two key features.

First, such information is often based on human perception, experience, and knowledge, and is typically expressed through language. These linguistic descriptions carry imprecision and vagueness, which makes them suitable for formalization using fuzzy sets.

Second, human perception, experience, and knowledge are not absolute sources of truth. Therefore, such information can only be trusted to a certain degree, and this trust is inherently partial. This partiality is also imprecise and can be formalized as a fuzzy value of a probability measure.

All of this highlights the necessity of creating a formal foundation capable of handling real-world information, with all its imprecision and uncertainty.

Z-numbers offer a framework for simultaneously representing both the imprecision of information and the degree of reliability associated with it. In this context, Z-rules—that is, “if-then” rules defined using Z-numbers—aim to capture both fuzziness and uncertainty in a unified manner. First proposed by L.A. Zadeh, this approach presents significant potential for real-world decision-making problems where information is often incomplete, vague, or partially reliable.

However, at present, there is no fully established and formal theoretical framework for approximate reasoning based on Z-rules. A rule base composed of Z-rules is considered complete only if, for every possible observation, there exists at least one rule whose antecedent (expressed as a Z-number) partially overlaps with the observation. Otherwise, the rule base is considered incomplete. This reflects a more realistic modeling assumption, as it is typically infeasible or impractical to construct a rule base that covers all possible scenarios involving fuzzy and probabilistic uncertainties.

When the Z-rule base is incomplete and a given observation is not matched by any rule, classical reasoning methods,such as those based on compositional rule of inference (e.g., Mamdani, Takagi-Sugeno),become ineffective in producing valid outputs. In such cases, implication-based approaches offer a more suitable alternative for reasoning with Z-rules, as they can better accommodate partial matches and incomplete knowledge.

Given the increasing complexity and uncertainty inherent in real-world systems, the development of a new formalism capable of deriving conclusions from Z-number-valued information is essential. Such a framework would lay the foundation for a new generation of approximate reasoning systems that can more effectively handle both imprecision and partial reliability, and thus enhance decision-making processes in fields such as expert systems, risk assessment, and intelligent control.

Statement of the problem

Below considers Z-conditional algorithm and development of software. Currently, there is almost no information in the scientific literature about Z-conditional reasoning, especially regarding logical inference based on Z-implication. Processing of Z-rules requires the use of a new type of implication. However, this problem has not been discussed in the scientific literature. The new type of implication, called Z-implication, is formed through the synergy of fuzzy and probabilistic implications (Aliev et.,2025). The related algorithm that enables inference using Z-valued If–Then rules is proposed. For simplicity let us consider reasoning for SISO model. Assume If-Then rules and current observation are given:

Rule i: If X is $\mathrm{Z_{ix}}(\mathrm{A_{ix}}, \mathrm{B_{ix}})$ Then Y is $\mathrm{Z_{iy}}(\mathrm{A_{iy}}, \mathrm{B_{iy}}) i = \overline{1, n}$,

Current observation: is $\mathrm{Z}'_{\mathrm{x}}(\mathrm{A'_x}, \mathrm{B'_x})$

The problem is to compute a Z-number-based value of Y: $\mathrm{Z}'_{\mathrm{y}}(\mathrm{A'_y}, \mathrm{B'_y})$

Proposed conditional reasoning algorithm.

The basic steps of the algorithm (Ahmadov,2025)] for solving this problem is described below:

Step 1. Using Z-implication (Aliev et.,2025) , compute a relation between Z- input $Z(A_{ix}, B_{ix})$ and Z-output $Z(A_{iy}, B_{iy})$ for each rule i=1,...,n.

A Z-implication $\mathrm{I_Z}$ may be described in terms of mapping between pairs of Z- sets $(\mathrm{A_1}, \mathrm{B_1}), (\mathrm{A_2}, \mathrm{B_2})$ taking into account underlying sets of cumulative probability distributions:

$$\mathrm{G_1} = \{\mathrm{p}: \int \mathrm{p_1} = \mathrm{p}, \sum \mathrm{p_1} \mu_{\mathrm{A_1}} \in \mathrm{B_1}\},$$

$$\mathrm{G_2} = \{\mathrm{q}: \int \mathrm{p_2} = \mathrm{q}, \sum \mathrm{p_2} \mu_{\mathrm{A_2}} \in \mathrm{B_2}\}.$$

Consequently, ALI-1 fuzzy implication and probabilistic implication would be a basis for formulating Z-implication I_Z as follows.

Z-implication I_Z is a vector-valued function:

$$I_Z = I_{FC}\left(k_1 I_F\left(\mu_{A_1}, \mu_{A_2}\right), k_2\{I_C(p, q): p \in G_1, q \in G_2\}\right),$$

where I_F and I_C are ALI-1 fuzzy implication and probabilistic implication respectively. We can get different probabilistic implications depending on copulas type. The second component of the vector-valued function $I_{FC} = (I_F(\mu_1, \mu_2), \{I_C(p, q)\})$ is a set of probabilistic implications $\{I_C(p, q)\}$ induced by sets of cumulative distributions G_1, G_2.

K is a binary two-dimensional column vector $K = (k_1, k_2)^T$, $k_1, k_2 \in \{0,1\}$. As special cases of Z-implication one has:

If $k_1 = 1, k_2 = 0$then I_Z is a fuzzy implication.

If $k_1 = 0, k_2 = 1$then I_Z is a set of probabilistic implications.

Z-implication is obtained in general case when $k_1 = 1, k_2 = 1$.

Step1.1. Set $k_1=1$, $k_2=0$, to apply fuzzy implication $I_{Fi}\left(\mu_{A_{ix}},\mu_{A_{iy}}\right)$ for each rule i=1,...,n.

Step1.2 Set $k_1=0$, $k_2=1$ to apply probabilistic implications $\{I_{Ci}(p,q): p\in G_{ix}, q\in G_{iy}\}$.

Step 2. Aggregate the computed fuzzy and probabilistic matrices of all the rules i=1,...,n:

Step 3. Perform composition operation of given current input $Z(A'_x, B'_x)$ and obtained aggregated matrices $(I_F,\{I_C\})$.

Step 4. Given A'_y and G'_y, compute B'_y to find $Z(A'_y, B'_y)$.

Python simulation

Numerical example.

1. If the error e is [negative small (NS), very sure] THEN the control action u is [negative small (NS),very sure].
2. If the error e is [zero (ZE),not very sure] THEN the control action u is [zero,not very sure].

If the error e is [positive small (PS),sure] THEN the control action u is [positive small (PS),not very sure].

$$not\ \ very\ \ sure=\left\{\frac{0.1}{0.6},\frac{1}{0.7},\frac{0.5}{0.8}\right\},\ sure=\left\{\frac{0.2}{0.75},\frac{1}{0.8},\frac{0.4}{0.9}\right\},\ very\,sure=\left\{\frac{0.3}{0.8},\frac{1}{0.85},\frac{0.3}{0.9}\right\}$$

$$e \text{ is } \left(\left\{\frac{0.09}{-10},\frac{0.15}{-7},\frac{0.32}{-3},\frac{0.59}{0},\frac{0.94}{3},\frac{0.71}{7},\frac{0.39}{10}\right\},\left\{\frac{0.2}{0.75},\frac{1}{0.8},\frac{0.4}{0.9}\right\}\right)$$

For this Z-number based input, find the Z-value of u.

```
numpy as np
import pandas as pd
def fuzzy_relation_matrix(X, Y, round_digits=3, as_dataframe=True):
    """
    Creates a fuzzy relation matrix R according to the rule:
    if x_i < y_j → 1 - x_i
    if x_i = y_j → 1
    if x_i > y_j → y_j
    """
    X = np.array(X, dtype=float)  # Convert X to a NumPy array of floats
    Y = np.array(Y, dtype=float)  # Convert Y to a NumPy array of floats
    R = np.zeros((len(X), len(Y)))  # Initialize a zero matrix of shape len(X) x len(Y)
    # Compute each element of the matrix
    for i, xi in enumerate(X):
        for j, yj in enumerate(Y):
            if xi < yj:           # If element of X is less than element of Y
                R[i, j] = 1 - xi   # Assign 1 - xi
            elif xi == yj:         # If elements are equal
                R[i, j] = 1        # Assign 1
            else:                  # If element of X is greater than element of Y
                R[i, j] = yj       # Assign yj
    if as_dataframe:  # Optionally convert to Pandas DataFrame
        R_df = pd.DataFrame(R,
                            index=[f"{X[i]}" for i in range(len(X))],  # Row labels from X
                            columns=[f"{Y[j]}" for j in range(len(Y))])  # Column labels from Y
        return R_df.round(round_digits)  # Round values to the specified number of digits
    else:
        return np.round(R, round_digits)  # Return NumPy array rounded to specified digits
def process_fuzzy_pairs(pairs, round_digits=3, as_dataframe=True):
    """
    Accepts a list of pairs (X, Y).
    For each pair, calls fuzzy_relation_matrix(X, Y)
    and prints the result.
    Arguments:
     pairs — list of tuples like [(X1, Y1), (X2, Y2), ...]
     round_digits — number of decimal digits for rounding (default 3)
     as_dataframe — if True, returns a Pandas DataFrame
    Returns a list of matrices (DataFrame or np.ndarray)
```

```
    """
    results = []  # Initialize a list to store results
    for idx, (X, Y) in enumerate(pairs, start=1):
        print(f"\n=== Matrix #{idx} ===")  # Print header for each matrix
        R = fuzzy_relation_matrix(X, Y, round_digits=round_digits, as_dataframe=as_dataframe)  # Compute fuzzy ma-
trix
        print(R)  # Print the matrix
        results.append(R)  # Store the matrix in results list
    return results  # Return all matrices
# Insert sets of fuzzy input values
A1 = [0.29, 0.56, 1, 0.68, 0.36, 0.15, 0.08] #input
A2 = [0.56, 0.81, 1, 0.87, 0.69, 0.45, 0.32] #output
A3 = [0.15, 0.29, 0.68, 1, 0.68, 0.29, 0.15]
A4 = [0.45, 0.62, 0.87, 1, 0.87, 0.62, 0.45]
A5 = [0.08, 0.15, 0.36, 0.68, 1, 0.56, 0.29]
A6 = [0.32, 0.45, 0.69, 0.87, 1, 0.81, 0.62]
# Insert pairs of sets for processing
pairs = [(A1, A2), (A3, A4), (A5, A6)]
# Compute fuzzy relation matrices for all pairs
```

Result of fragment of proposed algorithm are as follow:
Fuzzy relation matrixes:

```
          === Matrix #1 ===
      0.56  0.81   1.0  0.87  0.69  0.45  0.32
0.29  0.71  0.71  0.71  0.71  0.71  0.71  0.71
0.56  1.00  0.44  0.44  0.44  0.44  0.45  0.32
 1.0  0.56  0.81  1.00  0.87  0.69  0.45  0.32
0.68  0.56  0.32  0.32  0.32  0.32  0.45  0.32
0.36  0.64  0.64  0.64  0.64  0.64  0.64  0.32
0.15  0.85  0.85  0.85  0.85  0.85  0.85  0.85
0.08  0.92  0.92  0.92  0.92  0.92  0.92  0.92

          === Matrix #2 ===
      0.45  0.62  0.87   1.0  0.87  0.62  0.45
0.15  0.85  0.85  0.85  0.85  0.85  0.85  0.85
0.29  0.71  0.71  0.71  0.71  0.71  0.71  0.71
0.68  0.45  0.62  0.32  0.32  0.32  0.62  0.45
 1.0  0.45  0.62  0.87  1.00  0.87  0.62  0.45
0.68  0.45  0.62  0.32  0.32  0.32  0.62  0.45
0.29  0.71  0.71  0.71  0.71  0.71  0.71  0.71
0.15  0.85  0.85  0.85  0.85  0.85  0.85  0.85

          === Matrix #3 ===
      0.32  0.45  0.69  0.87   1.0  0.81  0.62
0.08  0.92  0.92  0.92  0.92  0.92  0.92  0.92
0.15  0.85  0.85  0.85  0.85  0.85  0.85  0.85
0.36  0.32  0.64  0.64  0.64  0.64  0.64  0.64
0.68  0.32  0.45  0.32  0.32  0.32  0.32  0.62
 1.0  0.32  0.45  0.69  0.87  1.00  0.81  0.62
0.56  0.32  0.45  0.44  0.44  0.44  0.44  0.44
0.29  0.71  0.71  0.71  0.71  0.71  0.71  0.71
```

2.To aggregate multiple fuzzy relation matrices obtained from different input pairs, a sequential combination procedure was implemented. The combination rule is defined as follows: if the sum of corresponding elements in two matrices is less than one, the value from the first matrix is retained; if the sum equals one, the resulting value is set to zero; and if the sum is greater than one, the value from the second matrix is used. This procedure is applied iteratively to all matrices, yielding a final aggregated fuzzy relation matrix.

```
=== R1 & ... & R3 ===
      0     1     2     3     4
5     6
0  0.92  0.92  0.92  0.92  0.92
0.92  0.92
1  0.85  0.85  0.85  0.85  0.85
0.85  0.85
2  0.45  0.64  0.32  0.32  0.32
0.64  0.32
3  0.45  0.32  0.32  0.32  0.32
0.62  0.32
4  0.45  0.45  0.69  0.87  1.00
0.81  0.32
5  0.32  0.45  0.44  0.44  0.44
0.44  0.44
6  0.71  0.71  0.71  0.71  0.71
0.71  0.71
```

The final step in the fuzzy relation analysis involves computing the A-vector using the Max–Min method. First, the matrix L is constructed by taking the minimum between each element of the input set X and the corresponding element in the aggregated fuzzy relation matrix R.Subsequently, the A-vector is obtained by taking the maximum across each column of matrix.

As result we had found values for A-part:

```
      y1     y2     y3     y4     y5
y6    y7
   Y  0.45   0.45   0.69   0.87   0.94
0.81  0.44
```

The next step, using B part of Z-number probability distributions are calculated using goal programming:

Objective function: $p = \sum_{i=1}^{7} \mu_i p_i$

$$\frac{\sum_{i=1}^{7} \mu_i x_i}{\sum_{i=1}^{7} \mu_i} = \sum_{i=1}^{7} x_i p_i$$

$$\sum_{i=1}^{7} p_i = 1$$

where $p_i \geq 0$, and points of obtained probability, x_i -universe points of A part of Z(A,B) number, μ_i -is membership degree on universe points on A, **p** is points of B part of Z-number/.

Obtained distributions on thepoints of B part of Z-number are as follow.

Obtained probabilities on input p=0.85 and on output p=0.85 are as follow:

p=0.85(input):((0,0.07564,0.567815,0.348348, 0.008197,0,0);

p=0.85(output):(0.164606, 0.313403, 0.276772. 0.217191, 0.028028, 0).

```
=== RULE 1 ===
        0.80      0.85      0.90      0.80      0.85      0.90
P1  0.000000  0.000000  0.000000  0.052445  0.000000  0.000000
p2  0.133333  0.075641  0.017949  0.175625  0.164606  0.046413
p3  0.507850  0.567814  0.694231  0.253120  0.313403  0.448053
p4  0.275965  0.348347  0.287821  0.229064  0.276772  0.364077
p5  0.082851  0.008198  0.000000  0.189929  0.217191  0.141457
p6  0.000000  0.000000  0.000000  0.099817  0.028028  0.000000
p7  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
-----------------------------------------------------------------
=== RULE 2 ===
         0.6       0.7       0.8       0.6       0.7       0.8
p1  0.022580  0.000000  0.000000  0.235321  0.110296  0.000000
p2  0.169310  0.100557  0.005219  0.142639  0.173011  0.180819
p3  0.215267  0.256807  0.301995  0.113935  0.172001  0.231693
p4  0.255312  0.328400  0.410549  0.093959  0.163373  0.243027
p5  0.165533  0.210650  0.258287  0.035800  0.119152  0.180114
p6  0.053266  0.064497  0.023950  0.065793  0.049696  0.074336
p7  0.118731  0.039089  0.000000  0.312553  0.212470  0.090012
----------------------------------------------------------------
=== RULE 3 ===
        0.75      0.80      0.90      0.60      0.70      0.80
p1  0.000000  0.000000  0.000000  0.275779  0.175553  0.052519
p2  0.000000  0.000000  0.000000  0.040485  0.045768  0.061870
p3  0.123583  0.065890  0.000000  0.067850  0.096793  0.146992
p4  0.271424  0.309886  0.287821  0.088371  0.135062  0.210834
p5  0.413968  0.490892  0.694231  0.000000  0.169435  0.261308
p6  0.191026  0.133332  0.017949  0.225122  0.186950  0.231467
p7  0.000000  0.000000  0.000000  0.302393  0.190437  0.035010
```

Result of aggregation on the computed probabilistic matrices is defined as follow:

```
R1agr:
          0         1         2         3         4         5         6
0  0.992473  0.992473  0.992473  0.992473  0.992473  0.992473  0.992473
1  0.620185  0.891592  0.891592  0.891592  0.891592  0.891592  0.891592
2  0.388061  0.494149  0.650150  0.609359  0.609359  0.609359  0.609359
3  0.187848  0.307431  0.452400  0.633700  0.708943  0.341792  0.341792
4  0.187848  0.307431  0.452400  0.589531  0.664774  0.795018  0.454341
5  0.187848  0.307431  0.452400  0.589531  0.664774  0.795018  0.706244
6  0.187848  0.307431  0.452400  0.589531  0.664774  0.795018  1.000000
R2agr:
          0         1         2         3         4         5         6
0  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
1  0.633148  0.941267  0.941267  0.941267  0.941267  0.941267  0.941267
2  0.348135  0.460674  0.684918  0.644430  0.644430  0.644430  0.644430
3  0.095283  0.223078  0.417144  0.665895  0.636811  0.315553  0.315553
4  0.095283  0.223078  0.417144  0.608880  0.777472  0.865698  0.412306
5  0.095283  0.223078  0.417144  0.608880  0.777472  0.865698  0.679696
6  0.095283  0.223078  0.417144  0.608880  0.777472  0.865698  1.000000
R3agr:
          0         1         2         3         4         5         6
0  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
1  0.333333  0.992277  0.992277  0.992277  0.992277  0.992277  0.992277
2  0.333333  0.409077  0.729084  0.660203  0.660203  0.660203  0.660203
3  0.017506  0.113874  0.389453  0.742087  0.664805  0.664805  0.664805
4  0.017506  0.113874  0.389453  0.662099  0.856391  0.958326  0.347300
5  0.017506  0.113874  0.389453  0.662099  0.856391  0.958326  1.000000
6  0.017506  0.113874  0.389453  0.662099  0.856391  0.958326  1.000000
```

In this stage, the SUMPRODUCT operation is performed between each incremental vector P and the A part vector. This involves element-wise multiplication followed by summation, resulting in a single scalar value for each P vector. These SUMPRODUCT values quantify the weighted contribution of each fuzzy result and serve as the final aggregated measures. Fragment of computer simulation listing and result are represented below:

```python
# Define the weight vector 'a' for the SUMPRODUCT calculation
a = np.array([0.45, 0.45, 0.69, 0.87, 0.94, 0.81, 0.44])

# === Function to compute SUMPRODUCT ===
def sumproduct(p_vector, a_vector):
    """
    Calculates the SUMPRODUCT between vector p and vector a.

    Args:
        p_vector (pd.Series): Incremental vector p (values to multiply).
        a_vector (np.array or list): Weight vector a.

    Returns:
        float: SUMPRODUCT result.
    """
    # Ensure both vectors have the same length
    min_len = min(len(p_vector), len(a_vector))

    # Compute element-wise product and sum
    return np.sum(p_vector.values[:min_len] * a_vector[:min_len])

# === Compute SUMPRODUCT for each incremental vector p ===
sumprod_p1 = sumproduct(p1, a)
sumprod_p2 = sumproduct(p2, a)
sumprod_p3 = sumproduct(p3, a)

# === Print results ===
print("SUMPRODUCT(p1, a) =", sumprod_p1)
print("SUMPRODUCT(p2, a) =", sumprod_p2)
print("SUMPRODUCT(p3, a) =", sumprod_p3)
# --- Description ---
# This function computes the SUMPRODUCT of an incremental vector p with a predefined weight vector a.
# The operation multiplies corresponding elements and sums them, producing a single numerical result .
```

Obtained result: $A_u =$

$\left(\left\{\frac{0.45}{-1},\frac{0.45}{-0.7},\frac{0.69}{-0.3},\frac{0.87}{0},\frac{0.94}{0.3},\frac{0.81}{0.7},\frac{0.44}{1}\right\}\right)$ and

$$B_u=\left(\left\{\frac{0}{0.539413},\frac{1}{0.6785440},\frac{0.3}{0.720213},\right).$$

Thus, $Z(A_u,B_u)$ is computed.

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YENI BIR ŞƏRTI MÜHAKIMƏ METODUNA DAIR

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Xülasə: Müasir mürəkkəb və məlumatlarla zəngin mühitdə qərar qəbuletmə sistemləri çox vaxt qeyri-müəyyən, qeyri-dəqiq və qismən etibarlı olan məlumatları effektiv şəkildə idarə etməlidir. Doğru və ya yanlış kimi ikili dəyərlər üzərində qurulmuş ənənəvi məntiq sistemləri bu cür real dünya mürəkkəbliklərini modelləşdirmək üçün qeyri-adekvatdır. Qeyri-səlis məntiq və ehtimal əsaslandırması qeyri-dəqiqliyi və qeyri-müəyyənliyi təmsil etməkdə irəliləyiş əldə etsə də, bu yanaşmalar adətən bu məsələləri ayrıca həll edir və qeyri-mükəmməl məlumatın ikili xarakterini -həm qeyri-dəqiqliyi, həm də etibarlılıq dərəcəsini tam əhatə edə bilmir. Bu yanaşma qeyri-dəqiqliyi və etibarlılığı vahid riyazi çərçivədə birləşdirir. İdarəetmə sistemləri, tibb, qərar qəbuletmə və məlumatların təhlili kimi sahələrdə nəzəri əsaslar və praktik tətbiqlər mövcud olsa da, Z qaydalarına əsaslanan təxmini nəticə çıxarış üçün tam işlənmiş və sistematik metodologiyalar qeyri-kafi olaraq qalır. Mövcud yanaşmalar çox vaxt bimodal məlumat əsaslı qayda bazalarının idarə edilməsində məhdudiyyətlərlə üzləşir. Bu çətinlikləri nəzərə alaraq, Z ədəd ilə qiymətləndirilən məlumatdan əsaslandırmaya imkan verən, daha insana bənzər anlayışı asanlaşdıran və qeyri-müəyyənliyin daha yaxşı idarə olunmasını təmin edən yeni formal modellərə ehtiyac var. Yuxarıdakılar mövcud qeyri-səlis və ehtimala əsaslanan implikasiya vasitələrinin təhlilini, onların çatışmazlıqlarının aradan qaldırılmasını vurğulayır və Z ədədi konsepsiyasına əsaslanan təxmini mühakimə üsullarının işlənib hazırlanması zərurətini əsaslandırır.

Təxmini mühakimə qərarların qəbul edilməsində və ekspert sistemlərində, xüsusən də məlumatın qeyri-dəqiq və qeyri-müəyyən olduğu situasiyalarda mühüm rol oynayır. Bu kontekstdə qeyri-səlis implikasiya modelləri qaydalara əsaslanan nəticələrin çıxarılması üçün əsas mexanizm kimi çıxış edir. Klassik məntiq “əgər... onda...” ifadələrini sərt şəkildə ifadə etdiyi halda, qeyri-səlis məntiq qeyri-səlis implikasiyalar vasitəsilə daha çevik və insana bənzər şərhə imkan verir.

Bununla belə, mövcud qeyri-səlis implikasiyalar - məsələn, Mamdani, Gödel, Lukasievicz, Zadeh, Aliyev, Reichenbach, Kleene-Dienes, Goguen, Yager, Weber, Fodor və başqaları tərəfindən təklif olunanlar praktik tətbiqlərdə məhdudiyyətlərlə üzləşirlər. Bu implikasiyalar yalnız qeyri-dəqiqliyi nəzərə alır, lakin məlumatın etibarlılığı və ya əminlik dərəcəsi kimi mühüm aspektləri adekvat şəkildə nəzərə ala bilmir.

Buna görə də, qeyri-səlis implikasiyanın funksional imkanlarını Z ədədinə əsaslanan implikasiyalara qədər genişləndirən yeni yanaşmalara artan ehtiyac var. Z ədədlər məlumatın qeyri-müəyyənliyini, həm də onun etibarlılığını (etibarlılıq və ya etibarlılıq) eyni vaxtda modelləşdirməyə imkan verir. Bu, daha məzmunlu və real düşünmə nəticələrinə gətirib çıxarır.

Bu işin aktuallığı ondan ibarətdir ki, o, mövcud qeyri-səlis implikasiya modellərinin çatışmazlıqlarını araşdırır, onların cari tətbiq potensialını təhlil edir və Z-ədədləri əsasında yeni implikasiyalaraın işlənməsini əsaslandırır. Bu, müasir qeyri-səlis mülahizə sistemlərinin inkişafı üçün həm nəzəri, həm də praktiki cəhətdən vaxtında və əhəmiyyətli tədqiqat istiqamətini təmsil edir. Yuxarıda qeyd olunan məqamlar işin aktuallığını müəyyən edir.

Qeyd etmək lazımdır ki, şərti nəticə çıxarma metodu idarəetmə sistemlərinin və qərar qəbuletmə sistemlərinin əsasını təşkil edir. Bu məqalədə qeyri-səlis və ehtimal implikasiyadan istifadə edərək yeni Z-şərti nəticə çıxarış metodu təklif edilir.

Açar sözlər: qeyri-səlis implikasiya, ehtimal implikasiyası, Z-təsirliliyi, ehtimal paylanması, Z ədədi, qeyri-səlis ədəd, yeni şərti nəticə çıxarış metodu.

О НОВОМ МЕТОДЕ УСЛОВНОГО РАССУЖДЕНИЯ

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Резюме: В современной сложной и насыщенной данными среде системам принятия решений часто необходимо эффективно обрабатывать информацию, которая является неопределенной, неточной и частично надежной. Традиционные логические системы, построенные на двоичных значениях, таких как «истина» или «ложь», неадекватны для моделирования таких сложных ситуаций реального мира. Хотя нечеткая логика и вероятностные рассуждения достигли прогресса в представлении неточности и неопределенности, эти подходы обычно решают эти проблемы по отдельности и не могут в полной мере охватить двойственную природу несовершенной информации — как ее неточность, так и степень надежности. Этот пробел стимулировал исследования Z-чисел и нечетких Z-соотношений; этот подход объединяет неточность и надежность в рамках единой математической структуры. Хотя теоретические основы и практические приложения существуют в таких областях, как системы управления, медицина, принятие решений и анализ данных, полностью разработанных и систематических методологий для приближенных рассуждений, основанных на Z-правилах, остается недостаточно. Современные подходы часто сталкиваются с ограничениями при работе с бимодальными базами правил, основанными на информации. Учитывая эти проблемы, существует потребность в новых формальных моделях, позволяющих делать выводы на основе информации, представленной в виде Z-чисел, что способствует более человеческому пониманию и более эффективному управлению неопределенностью. Вышеизложенное освещает анализ существующих инструментов нечеткой и вероятностной импликации, устранение их недостатков и обосновывает необходимость разработки методов приближенной импликации, основанных на концепции Z-чисел.

Приближенные рассуждения играют решающую роль в принятии решений и экспертных системах, особенно в ситуациях, когда информация неточна и неопределенна. В этом контексте модели нечеткой импликации служат ключевым механизмом для вывода заключений на основе условных правил. В то время как классическая логика выражает утверждения «если..., то...» жестко, нечеткая логика обеспечивает более гибкую и человеческую интерпретацию посредством нечетких импликаций. Однако существующие нечёткие импликации, такие как предложенные Мамдани, Гёделем, Лукасевичем, Заде, Алиевым, Райхенбахом, Клин-Диенесом, Гогеном, Ягером, Вебером, Фодором и другими, сталкиваются с ограничениями в практическом применении. Эти импликации учитывают только неточность, но не учитывают в полной мере такие важные аспекты, как надёжность или степень достоверности информации.

Поэтому растёт потребность в новых подходах, расширяющих функциональные возможности нечёткой импликации, распространяя её на импликации на основе Z-чисел. Z-числа позволяют одновременно моделировать как нечёткость информации, так и её достоверность (надёжность или достоверность). Это приводит к более выразительным и реалистичным результатам рассуждений.

Актуальность данной работы заключается в том, что она исследует недостатки существующих моделей нечёткой импликации, анализирует их текущий прикладной потенциал и обосновывает разработку новых импликаций на основе Z-чисел. Это актуальное и значимое направление исследований, как с теоретической, так и с практической точки зрения, для развития современных систем нечеткого вывода. Вышеперечисленные моменты определяют актуальность данной работы.

Следует отметить, что метод условных выводов лежит в основе систем управления и принятия решений. В данной статье предлагается новый метод Z-условного вывода, использующий нечеткую и вероятностную импликацию.

Ключевые слова: нечеткая импликация, вероятностная импликация, Z-импликация, распределение вероятностей, Z-число, нечеткое число, новый метод условного вывода.