THE ROLE OF CRITERIA IN PORTFOLIO SELECTION PROBLEM: COMPARATIVE ANALYSIS OF CLASSICAL AND FUZZY APPROACHES

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Received: *12 May 2026* ***Revised:*** *4 June 2026* ***Accepted:*** *25 June 2026*

UOT: 658.148 **JEL:** D81 **DOİ:**https://doi.org/10.32010/CIBZ4637

Abstract: Purpose: The aim of this paper is to explore the identification, structuring and evaluation of the criteria for portfolio selection in an uncertain environment. It analyzes shortcomings of the traditional mean-variance framework - especially the lack of information, multidimensional risk, and multiple decision criteria, and explores how fuzzy extensions of this framework can enhance classical approaches rather than replace them.
Method: The method is based on research in the domains of multi-criteria portfolio choice, interactive decision support, fuzzy portfolio evaluation and results in the development of a comparative analysis between the traditional and the fuzzy approaches. The process of portfolio selection is divided into two stages: 1) the identification of efficient portfolios and 2) the ranking of efficient portfolios according to a coherent system of measures that includes the expected return as well as differentiated risk measures, diversification features and other financial metrics.
Findings: Classical models have analytical superiority and computational transparency, while fuzzy models offer a higher level of flexibility and careful consideration of uncertainty, especially in cases of market instability, imperfect information and expert judgment. However, both are not mutually exclusive, oppositely they are dealing with two different levels of the same decision problem. The conclusion of the article is that the choice of methodology of optimization is not the only factor affecting the quality of the portfolio - the identification, organization, weighting, and interpretation of assessment criteria are also important.
Keywords: portfolio selection, mean-variance model, fuzzy approach, efficient frontier, multi-criteria decision making

Introduction

The selection of a portfolio is a critical part of financial decision making considering trade-off between investment goals and uncertainty, resource allocation, and risk tolerance. In the traditional framework the process is considered as a choice of the portfolio that maximizes expected return for a given level of risk or minimizes risk for a given level of required return. The most popular analytical framework that can be used in this problem is the mean-variance approach. It introduced diversification and put the concept of efficient frontier into practice in addition to the basis for comparison of different portfolios [1, 2]. However, in real-world conditions investment portfolio selection is not restricted with the developing efficient frontier. The next step is for investors and managers to determine which of the available portfolios is the most reasonable – and that choice is more likely to be influenced by a variety of factors rather than uniformed by risk and return.

This assumption is supported with the relevant studies. Once risk is considered multidimensional and it is turned into risk criteria that are understandable to real investors, portfolio support becomes more practical, argues Hurson and Ricci-Xella [3]. Similarly, Polyashuk [4]

presents portfolio selection as a real multiple criteria problem, and divides the criteria into those that describe individual portfolio components and those that describe the portfolio as a whole. These contributions focus the discussion from optimization alone to the preceding question: how the relevant criteria must be formulated and structured. The author has done some previous research which is helpful for the present article. Hasanova [5] showed the possibility of building fuzzy efficient frontier if the return and risk are represented with fuzzy value with the inclusion of historical information and expert judgment. In a direct comparison, Hasanova [6] remarked that the fuzzy formulation provides interval valued return and risk information and possible false precision of crisp estimates is avoided. The question raised in the present article is: how do the criteria of portfolio selection get identified, evaluated and interpreted in an uncertain environment?

Thus, the paper constructs a criteria-based analysis of portfolio choice in the context of uncertainty. The main idea is that the choice of the portfolio should be considered as a two-stage process. First, a set of feasible or efficient portfolios are formulated. Secondly, the portfolios are assessed and chosen according to a carefully developed set of criteria that have to be explained, weighted, and interrelated. From this point of view, the classical and fuzzy methods are not precluding one another, they are dealing with different levels of the same decision problem.

1. Classical portfolio theory as the analytical foundation

The first step is to adopt classical portfolio theory as the analytical basis due to its analytical clarity. Portfolio return is treated as a weighted average of the expected returns of the securities comprising portfolio and portfolio risk is measured by variance or standard deviation, which considers variability of securities and correlation between securities. The efficient frontier is defined as the combination of portfolios that offer the maximum expected return for a given level of risk. This is necessary as a simple mathematical standard for the more elaborate approaches to compare. Many research studies continue to use this baseline even if the main goal is not the classical optimization. Biswas et al. [7] for example, in their study, focus on performance measures like annualized return, the Sharpe ratio, Sortino ratio, information ratio, alpha, beta, and R-squared for a staged approach to mutual fund selection. The persistent presence of these indicators confirms the ongoing relevance of mean-variance and CAPM argumentation in a portfolio analysis. Similarly, Derya et al. [8] build their intuitionistic fuzzy models directly on the Konno-Yamazaki MAD formulation [9], which preserves the traditional risk-minimization objective in linear form while extending the return constraint to an intuitionistic fuzzy number representation - a choice that demonstrates how advanced fuzzy methods reinforce rather than discard the classical structure.

However, the limitations of the classical approach are obvious. It is difficult to follow unconditionally the assumption that expected returns are forecasted based on historical observations, and covariance are treated as stable guides to future behavior. The market environment fluctuates, firm-specific prospects evolve, macroeconomic shocks influence the behavior of assets, and some information relevant to the future performance incorporated to the decision making process due to managerial interpretation or partial qualitative assessment. Once this is recognized, the portfolio selection problem is no longer a purely mechanical optimization of fully known values. Thus, the classical framework is best viewed as an analytical baseline rather than a complete decision framework.

1.1. Designing a well-structured set of criteria

The argument that portfolio selection cannot be limited to an undifferentiated list of metrics is an important contribution of the multiple-criteria related literature. Decision criteria differ not only in content but also in logical role. Polyashuk [4] suggests aa especially useful distinction by dividing criteria into two groups: criteria specifying both the entire portfolio and its individual items, and those that applicable only to the portfolio as a whole. This separation prevents the decision maker from handling every criterion as operating at the same decision level.

Since both individual assets and portfolios have return and risk properties, expected return and several risk indicators referred to the first

group. By contrast, diversification features may be considered as relevant only at the portfolio level. Thus, determination of criteria in portfolio selection problem requires a coherent structure that reflects the economic meaning and decision role of each criterion.

This assumption was reinforced by Hurson and Ricci-Xella [3] from a different perspective. They demonstrate that risk is not conceptually homogeneous: there are common risks across the whole market and residual risks, but these two types of risks play different roles in portfolio evaluation. Moreover, if each common-risk component is included separately into the decision model, then its may outweigh others simply through numerical multiplication. The solution provided by authors is a synthesis criterion for common risk and a trichotomic assessment approach consisting of expected return, common risk, and residual risk. The implication of the methodology is unambiguous: the creation of a valid criterion system requires that its conceptual symmetry be designed, not spontaneously achieved through the addition of metrics.

Addressing abovementioned to the present topic, criteria for portfolio selection under uncertainty may be classified into four coherent groups. The first group includes core financial criteria- principally expected return and broad risk measures. The second contains risk-structure criteria, such as common versus residual risk or systematic versus specific exposure. The third includes portfolio-structure criteria, including diversification, the number of securities, and sector balance. The fourth group contains supplementary financial and market indicators that may guide pre-selection or refined ranking. This organization maintain the primary basis while acknowledging that portfolio selection process is significantly broader than a standard bi-criteria optimization.

2. An importance of criteria determination

The subsequent step following the identification of portfolio selection criteria is determination of their relative importance. Throughout this process, a clear logic of weighting is established, rather than following a linear sequence. In the Polyashuk framework [4], the first set of criteria is more important than the second set. This assessment should begin with the "primary" criteria domain. After that, a “ordered” set of criteria is used for the final selection of the portfolio. This means that not all criteria will balance each other out.

Staged empirical methods illustrate a similar idea. Biswas et al. [7] suggest a two dimensional approach, that is, screening of mutual funds using Data Envelopment Analysis (DEA) and then rank the funds using MABAC that considers entropy-based criteria weights. Although this framework focuses on mutual fund selection rather than direct stock portfolio selection, the methodological approach is quite relevant: certain criteria more applicable for elimination or screening, while others are better positioned for comparative ranking. Merging everything into a single undifferentiated step is adversely influencing the conceptual clarity of the model.

An importance of each criterion should be defined based on three considerations at least. The primary consideration is conceptual centrality: criteria that reflects the investment goal. The second is reliability of measurement: criteria that are crucial and highly relevant should not become a decisive criterion, if they are estimated inaccurately. The third one is the decision context: long-term strategic capital allocation, tactical positioning and conservative capital preservation strategies are not imply similar priority. While uniform weighting is convenient from a computational standpoint, it is often not justified as a general rule.

The weighting of the components of a criterion should not be carried as a purely statistical operation added on to the end of the substantive design stage. It is a decision-making act in itself, and is important objective procedures for weighting criteria can be supplemented by expert judgment regarding their hierarchy and importance in uncertain environment. In this context, determining the importance of criteria is an integral part of identifying criteria themselves.

2.1. Assessment of criteria in an uncertain environment: Application of fuzzy approaches

The most substantial argument for fuzzy portfolio approaches is not that classical portfolio optimization is incorrect, but that many of the portfolio metrics cannot be properly expressed using crisp measures. Both classical and fuzzy approaches have been applied to the same eight

asset portfolio problem, previously, by the author in a comparative study. The crisp formulation resulted in a set of point-valued efficient portfolios and the fuzzy formulation, in an interval-valued return and risk for each efficient portfolio and a fuzzy efficient frontier with core and bounds. The comparison demonstrated that the fuzzy model has more and cautious presentation of the uncertainty estimation [5, 6]. This difference relates to the status of the criteria outlined and it is not merely a matter of calculation. Although there is clear observation of variance estimates, covariances and historical returns, even using their values as estimates in the future generates uncertainty. Even some of the things that impact portfolio selection importance like future resilience assessments of the company or the quality of the data supporting expectations are subjective. Hasanova [5] and Pośpiech [8] agree that fuzzy representations enable the inclusion of uncertain or expert-based evaluations into the decision model. In order to get complementary evidence, Pośpiech [8] compares the results of the fuzzy TOPSIS (FTOPSIS) with the outcomes of the standard TOPSIS in the context of the companies belonging to WIG20.

Evaluation criteria are fundamental and market indicators such as return on assets (ROA), return on equity (ROE), price/Book Value (P/BV) and price/Equity (P/E) are considered. Multi-year values are represented as triangular fuzzy numbers with their lower, middle and upper parameters representing the pessimistic, expected and optimistic estimates respectively. For each of the pre-selected portfolios the classic Markowitz framework is used to create the portfolio. But the logic is important: there does not need to be a "crisp average" when it comes to criterion evaluation. In fuzzy representation, when the informational environment is either unstable or multi-period, the nuance that is relevant to decision-making is preserved. The choice of a well-balanced position is better than methodological narrow-mindedness. However, not all criteria have to be fuzzified. For instance, budget share, realized historical values may be considered in crisp representation. Others require to be dealt with as interval-valued, fuzzy, or judgment sensitive. Therefore, a mixed framework is the most suitable framework under uncertainty: crisp- for well-observed and stable criteria, and fuzzy for imprecise and confidence dependent.

3. Generation of the efficient frontier and the final selection of the portfolio.

A major limitation of the conventional portfolio analysis is the transition from a set of efficient portfolios to the choice of a single portfolio for implementation. Another major concern is that the efficient frontier only shows feasible alternatives and will not establish which is the best option accordingly the number of other considerations, including additional criteria, preference structures, and uncertainty. An illustration of this assumption is provided by Hurson and Ricci-Xella [3]. They first estimate the return on the portfolio, identifying the risk sources, using Arbitrage Pricing Theory (APT), then they use ELECTRE TRI to select a synthesis criterion from a multitude of common risk dimension and finally, they use MINORA as an interactive tool for choosing an appealing portfolio. The method is not just a extension of the frontier analysis, the method is a complete route from generated alternatives, through a considered final decision. The similar reasoning is observed in other staged frameworks that do not "combine" screening, weighting, and ranking into a single action.

The classical and fuzzy approaches are also linked by means of this bridge. Classical optimization has been very effective with structuring the feasible set and expressing the efficient frontier, while the use of fuzzy and multi-criteria techniques is most valuable when uncertainty, certainty level and the importance of the criteria are variable. The efficient frontier is essential but it is not complete- the geometry applies to what portfolios can be, but not to how to select the most suitable accordingly to preferences. Therefore, any extensive treatment of portfolio selection criteria should not be just a list of metrics, or a frontier graph- it should explain how each of these criteria leads from the optimization output to a choice among the various possibilities. The contrast between classical and fuzzy can be now precisely defined. The classical method is analytical transparent, easily computed, and interprets the return-risk trade-off explicitly. It is still an essential foundation for the construction of efficient frontiers and for defining the set of feasible portfolios. The fuzzy approach provides flexibility in the representation

of uncertain criteria, allows expert judgement and avoids the contamination of the precision that can take place in the representation of volatile estimates.

To summarize, the two frameworks cannot be viewed as alternatives to each other. They, on the other hand, treat different facets of the same kind of decision problem. Understand importance of classical optimization for generating and organizing feasible portfolios, and utility of the fuzzy extensions and multi-criteria approaches when the analyst has to discover which criteria are important, how they are defined, how they should be grouped together or weighted and how one of the various efficient portfolios should be evaluated under imperfect information. Fuzzy approach does not reject the classical portfolio theory, rather it only enhances conditions of its applicability.

Practical Implications and Directions for Further Research

The main implication for researchers is methodological. Portfolio selection studies may make it explicit in their approach both the explicit decision modeling used as well as the manner in which the criteria were organized and how they were prioritized. This should separate criteria that are considered to be stable observations versus criteria that involve some uncertainty. With that lack of transparency it is impossible to make comparisons between the alternative portfolios because the varying outputs could be more a reflection of what criteria are being used in the models than on the optimization procedure. For practitioners, the results are no less tangible: If the portfolio manager uses classical risk-return estimates alone, the clarity of those estimates is an advantage but they leave out the impact of residual risk, common risk, diversification structure and supplementary indicators on the attractiveness of efficient portfolios. On the other hand, if a manager adopts explicitly advanced multi-criteria fuzzy methodology without maintaining the simplicity of the classical one, decision-maker may end up losing this interpretability. The proposed integrated approach is a solution between two extremes: over-simplification on one hand, and on the other hand unnecessary methodological fragmentation. It is natural that several new domains for research arises from it. A first step would be to conduct additional research on the criterion hierarchies that could be comprehensible even for investors but still sufficiently analytical. Second, comparative studies should also assess the impact of various ways of structuring the same criterion (crisp version, interval version, fuzzy version, reliability weighted version) on the final rank of feasible portfolios.

Findings

The analysis outlined in this paper leads to three main conclusions. Although expected return and risk continue to be the basic analytical concept, in the portfolio selection process these measures should not be used as the sole criteria in portfolio selection process. The problem of application is broader: the criteria vary with respect to economic substance, logical structure, measurement accuracy and investor sensitivity. Determination of the proper criterion is a primary task, not secondary technique adjustment. Secondly, there must be coherence in the evaluation of the criteria. It is conceptually useful to make a distinction between criteria that apply to an asset and to a portfolio, and between common risk and residual risk. These frameworks illustrate that a proper portfolio-support system depends less on the number of metrics, than on the coherence with which they organized and interpreted.

Thirdly, uncertainty alters the assessment of criteria. In classical mean-variance framework, it is the starting point, but when the criteria cannot be represented by a crisp estimate the fuzzy extensions will come into play. In such cases, the fuzzy techniques enhance the performance of the model and make the choice realistic. The quality of the decision, however, depends not only on optimization algorithm but also on the identification, grouping, weighting and proper interpretation of selection metrics under uncertainty.

To conclude, findings suggest an integrated framework for portfolio choice. The feasible set should be may be generated and structured using classical optimization. When uncertainty and lack of information are important and the role of the different criteria is not clearly determined, fuzzy and multi-criteria techniques should help in the selection and final assessment of portfo-

lios. Such a framework is theoretically more balanced, and more likely to reflect actual conditions of investment decisions.

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PORTFEL SEÇİMİ PROBLEMİNDƏ MEYARLARIN ROLU: KLASSİK VƏ QEYRİ-SƏLİS YANAŞMALARIN MÜQAİSƏLI TƏHLİLİ

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Xülasə: Tədqiqatın məqsədi: Məqalənin məqsədi qeyri-müəyyənlik şəraitində investisiya portfelinin seçim meyarlarının müəyyən edilməsi, strukturlaşdırılması və qiymətləndirilməsi məsələlərinin tədqiqidir. Ənənəvi "orta-dəyişkənlik" modelinin məhdudiyyətləri- natamam məlumat, çoxölçülü risk və müxtəlif qiymətləndirmə meyarlarının nəzərə alınmaması baxımından araşdırılmışdır. Qeyri-səlis genişləndirmələrin ənənəvi yanaşmaları əvəz etməkdən daha çox, onların nəzəri-metodoloji əsaslarını tamamlayaraq qeyri-müəyyənlik şəraitində tətbiq imkanlarını necə genişləndirdiyi araşdırılmışdır.

Tədqiqatın metodu: Tədqiqat çoxmeyarlı portfel seçimi, interaktiv qərar dəstəyi və qeyri-səlis portfel qiymətləndirilməsi sahələrində mövcud elmi ədəbiyyata əsaslanır və ənənəvi yanaşmalarla qeyri-səlis metodların müqayisəli təhlilini əhatə edir. Bu çərçivədə portfel seçimi strukturlaşdırılmış iki mərhələli proses kimi nəzərdən keçirilir. Birinci mərhələdə mümkün və ya səmərəli portfellər müəyyənləşdirilir, ikinci mərhələdə isə həmin portfellər gözlənilən gəlirlilik, differensiallaşdırılmış risk ölçüləri, diversifikasiya xüsusiyyətləri və əlavə maliyyə göstəricilərini özündə birləşdirən ardıcıl meyarlar sistemi əsasında qiymətləndirilir.

Tədqiqatın nəticələri: Təhlil göstərir ki, ənənəvi modellər analitik dəqiqlik və hesablama şəffaflığı baxımından öz elmi-praktiki əhəmiyyətini qoruyub saxlayır. Qeyri-səlis yanaşmalar isə xüsusilə bazar qeyri-sabitliyi, natamam informasiya və ekspert mühakiməsinin nəzərə alınmasının zəruri olduğu hallarda daha yüksək çeviklik təmin edir və qeyri-müəyyənliyin daha dəqiq qiymətləndirilməsinə imkan yaradır. Bu baxımdan, hər iki

yanaşma bir-birini istisna etmir - əksinə, eyni qərar qəbuletmə məsələnin müxtəlif səviyyələrini tamamlayır. Tədqiqatın nəticələrinə əsasən, portfel seçiminin keyfiyyəti yalnız tətbiq olunan optimallaşdırma metodundan deyil, həm də qiymətləndirmə meyarlarının düzgün müəyyənləşdirilməsindən, sistemləşdirilməsindən, çəkiləndirilməsindən və təfsir olunmasından əhəmiyyətli dərəcədə asılıdır.
Açar sözlər: portfel seçimi, orta-dəyişkənlik modeli, qeyri-səlis yanaşma, səmərəlilik sərhədi, çoxmeyarlı qərar qəbuletmə

РОЛЬ КРИТЕРИЕВ В ЗАДАЧЕ ВЫБОРА ПОРТФЕЛЯ: СРАВНИТЕЛЬНЫЙ АНАЛИЗ КЛАССИЧЕСКИХ И НЕЧЕТКИХ ПОДХОДОВ

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Резюме: **Цель исследования:** В статье исследуется проблема формирования, систематизации и оценки критериев выбора инвестиционного портфеля в условиях неопределенности. Особое внимание уделяется ограничениям традиционной модели «средняя доходность — дисперсия», которая, несмотря на свою методологическую значимость, не в полной мере отражает практические условия принятия инвестиционных решений. В частности, данная модель ограниченно учитывает неполноту исходной информации, многомерную природу риска и необходимость одновременного согласования нескольких, нередко конкурирующих, критериев. В этой связи рассматривается потенциал нечетких расширений традиционного подхода как инструмента его дополнения и методологического развития, а не замены.
Метод исследования: Исследование опирается на научные работы в области многокритериального выбора портфеля, интерактивных систем поддержки принятия решений и нечеткой оценки инвестиционных альтернатив. На этой основе проводится сравнительный анализ традиционных и нечетких подходов к выбору портфеля. В статье выбор портфеля рассматривается как структурированный двухэтапный процесс. На первом этапе определяется множество эффективных портфелей, на втором - осуществляется их оценка на основе согласованной системы критериев, включающей ожидаемую доходность, дифференцированные показатели риска, характеристики диверсификации, а также дополнительные финансовые индикаторы.
Результаты исследования: Проведенный анализ свидетельствует о том, что традиционные модели по-прежнему сохраняют научную и практическую значимость, поскольку опираются на четко формализованный механизм, обеспечивают аналитическую строгость и обеспечивают прозрачность вычислительных процедур. В то же время нечеткие подходы расширяют возможности анализа инвестиционного портфеля, обеспечивая более гибкий и взвешенный учет неопределенности, особенно в условиях нестабильности рынка, неполной информации и необходимости учета экспертных оценок. Традиционные и нечеткие подходы не следует рассматривать как взаимоисключающие- они раскрывают различные аспекты одной и той же задачи принятия инвестиционных решений. В статье обосновывается вывод о том, что качество выбора портфеля определяется не только применяемым методом оптимизации, но и степенью обоснованности выявления, структурирования, взвешивания и интерпретации оценочных критериев.
Ключевые слова: выбор портфеля, модель «средняя доходность — дисперсия», нечеткий подход, граница эффективности, многокритериальное принятие решений.